Mathematics, English for Sek I and Sek II

Mathematica - The Principles of Math

15. Geometrical Construction, the Fruit of persistent Research

10:12 minutes

00:25 The ancient Egyptians, tasked with measuring the area of agricultural land around the Nile, used a variety of tools, including rope, poles, and rulers. Their calculations and measurements were very accurate and their skills were unrivaled.

00:41 Thanks to their skill at measuring, they were able to construct some of the grandest architectural marvels the world has seen, the Pyramids.

00:59 Yet when it comes to geometry, Greece is the first place that comes to people's minds.

01:01(caption) Greece

01:06 The ancient Greeks took over the Egyptians' measurement skills and explored the structure of diagrams as an area of academic research instead of practical application.

01:17 Drawing diagrams with circles and straight lines was an interesting pastime and academic exercise that brought amusement to Greek scholars.

01:27 There was an important rule for their game. They must use a ruler without markings and a foldable compass.

01:47(caption) Oh, No!

01:42 It was because the ancient Greeks believed that exploring the nature of diagrams by use of simple devices was more valuable than measuring their actual length.

01:56 (caption) "Adopting more sophisticated and complex devices is worthless manual labor to a philosopher like me."
– Plato (c. 429 BC – c. 347 BC)

02:09 Geometrical constructions had an influence on the book Elements of Geometry, the seminal work of Euclid, the so-called father of Greek geometry.

02:19 This important text explained the five basic rules called the five postulates of Euclidean geometry. Among them, we discover three rules on geometrical construction.

02:22 1. A straight line can be drawn from any point to any point.
2. A straight line can be extended indefinitely.
3. A circle may be described with any given point as its center and any distance as its radius.

03:02 Okay, then how about constructing some diagrams according to

these rules?

03:07 (caption) 1. Constructing a perpendicular line at the midpoint of a line segment

03:16 First, we draw a circle with point A as its center, and then draw another circle with the same radius with point B as its center.
Do you see the two points where the circles meet?
The line connecting these two new points is the perpendicular at the midpoint of AB.

03:34 (caption) 2. constructing a bisector of an angle

03:34 Now we're going to construct the bisector of an angle. In other words, cutting the angle in two.

03:42 First, we draw a circle with center O at the point where lines X and Y meet. This circle meets X and Y at points A and B.
Using points A and B as a center, draw two arcs with the same radius. These arcs meet at point P. If we connect points O and P with a line, that line is a bisector of the original angle XOY.

04:12 Next, let's construct two equal angles.

04:16 (caption) 3. constructing two equal angles

04:22 To construct two equal angles, let's first draw part of a circle with a center at O, and then part of another circle with the same radius with its center at P.
If we determine the distance between points A and B in angle XOY and then use that distance to draw part of a circle with a center at C, we will create point D.

04:46 If we connect P and D, it completes the angle equal to the first.

04:54 By using these methods, we're able to construct much more complicated diagrams.

05:01 (caption) 1. regular triangle
2. regular hexagon
3. circumcenter of a triangle
4. regular pentagon inscribed in a circle

06:08 How about that? See how many complicated diagrams we can make using just a compass and a straight edge?

06:19 Thanks to all these efforts, Greek geometry evolved tremendously.

06:24 But there were three unsolved diagrams which the Greeks struggled to construct.

06:34 (caption) 1. Can you construct a cube with double the volume of a random cube?
2. Can you equally divide a random angle by three?

3. Can you construct a square the same area as a specific circle?

07:20 (caption) 3 BC – 1882 AD (almost two thousand years)

07:11 Those three questions puzzled mathematicians for nearly two millennia. The answer to these questions was not determined until the nineteenth century.

07:29 Thanks to many mathematical interpretations, the conclusion was that these three questions can't be resolved using traditional Greek geometrical constructions.

07:42 What if we simply use rulers and protractors that have markings?

07:39 Maybe those questions could have been solved more quickly and simply with a few more complex tools.

07:47 It might look like a frustrating exercise, but these efforts to construct diagrams without using marked divisions resulted in great geometric figures, including the ellipse, parabola, hyperbola, cubic curves, and quadratic curves.

08:17 Modern-day humans living in high-tech society rarely use ancient Greek geometric construction directly in their daily lives…

08:25 But as the three unsolved geometrical constructions contributed so much to the evolution of math, Greek geometry has made tremendous contributions to a number of fields, including modern architecture, painting, and design.

08:39 Nearly every famous work of architecture or sculpture adopted the golden ratio for balance.

08:45 The ancient Greeks employed an exact "golden ratio" in their subjects through geometrical construction methods, which had a profound influence on modern architecture and design.
French mathematician Gaspard Monge was able to express three-dimensional objects in two dimensions using geometrical diagrams, which helped answer many questions in construction.

09:08 This method was called descriptive geometry, which today is a major component of constructional designing and planning.

Gaspard Monge (1746 – 1828)

09:21 A method of perspective, which expresses three-dimensional volume on flat surfaces, was based on mathematical theory, developed from projective geometry which had evolved from descriptive geometry.

09:35 (caption) Simple devices, stubborn principles…

09:41 Geometrical construction contributed greatly to human history and civilization.