Mathematics, English for Sek I and Sek II

Mathematica - The Principles of Math

17. The Principles of Enlargement and Reduction, Similarity in Diagrams

09:57 minutes

00:31 “Wow, you’re a chip off the old block.” Is this something you’ve heard a lot?

00:39 Being similar to your mom or dad is very different from the similarity we find in math. This is about similar diagrams, which are the basic principles for enlargement and reduction.

00:54 There are strict guidelines for defining similarity in math.

00:55 (caption)
diagram similarity
If a diagram is enlarged or reduced on an equal scale, it should be
When a diagram is enlarged or reduced on a regular scale, it must remain congruent with its original shape. This is called similarity. And this is the math symbol of the similarity.

01:00 When a diagram is enlarged or reduced, it must remain congruent with its original shape. This is called similarity. And this is the math symbol for similarity.

01:12 math symbol for similarity

01:14 What if it is expanded in one direction or part of the diagram is reduced?

01:20 In math, if a diagram is considered similar to another, the entire diagram must be enlarged at a regular ratio and it must be in congruence with its original shape.

01:31 Otherwise we wouldn’t consider two diagrams as similar even if they looked very similar.

01:45 Two unique characteristics are found in similar diagrams.

01:50 Let’s measure the length of the sides of two figures.

01:56 You will find the same ratio between the lengths of corresponding sides on each diagram.

02:24 (caption) i. The ratio of the lengths of corresponding sides is constant.

02:21 In similar diagrams, the ratio of the lengths of corresponding sides is always constant.

02:27 This ratio is called the ratio of similarity.

(caption)
ratio of similarity (similitude)

***Both terms are used, so let's keep both in the caption, even if we use just one (similarity is much more common).

02:34 Another unique characteristic is found in corresponding angles.

02:38 Corresponding angles in each figure are the same size.

03:07 (caption) ii. The size of corresponding angles is the same in two similar diagrams.

03:12 Just as with two-dimensional figures, similarity is found in three-dimensional figures.

03:17 (caption) characteristics of three-dimensional similar figures

03:20 Similar figures have a constant ratio of the lengths of their corresponding sides.

i. Corresponding sides have a constant ratio of their lengths

03:29 Corresponding sides of similar objects are themselves similar figures.

ii. Corresponding sides are similar figures.

03:41 Can we say a ball used in rhythmic gymnastics and a ping-pong ball are similar?

03:47 Let's consider a circle, which of course has no angles or sides.

03:53 Its size depends on the length of its radius. It always maintains a constant, round shape.

04:01 Therefore, every circle in the world is similar and their ratio to each other can be expressed using the ratio of their radiuses.

04:11 The same rule applies to a sphere, a three-dimensional figure. A ping-pong ball and a ball used in rhythmic gymnastics are similar as long as they don't have any indentions.

04:24 This is a pyramid, a symbol of ancient Egypt. The largest of them reaches up to 147 meters in height.

04:36 (caption)
Thales of Miletus (624 BC – 546 BC)
early Greek philosopher, established the foundations of geometry

04:32 The Greek mathematician Thales of Miletus, a teacher of Pythagoras, used a single rod to determine the height of a pyramid, something believed impossible to be measured.

04:43 What made the measurement possible was similarity of shapes.

04:49 On a sunny day, if you set up a stick next to the pyramid, …
… the direction of the sun's rays, the stick, and the shadow create a

right triangle.

05:00 Another right triangle is created next to the pyramid.

05:04 Since both right triangles are in virtually the same spot at the same time, the angle of elevation of the sun is also the same. The angle between the surface and the height is a right angle on both triangles. That means the two right triangles are similar.

05:19 (caption)
a: length of the stick's shadow
h: length of the stick
a′: length of the pyramid's shadow
h′: height of the pyramid

height of the pyramid (h′) = a′*h/a

05:22 In accordance with the characteristics of similar figures, if we measure each shadow's length and the length of the stick, we can calculate the proportional expression. This gives us the height of the pyramid.

05:33 height of the pyramid (h′) = a′*h/a

05:42 This is a sheet of A4-sized paper, the most frequently used size of paper in the world.

05:47 The standards for paper are determined when the paper mill cuts a large piece. A4-sized paper is created by cutting A0-sized paper four times.

05:59 If we compare the length of the sides, they all have a constant ratio.

06:24 If we print using sheets of paper that are not similar but enlarged or reduced, paper standards have to be adjusted each time. That would lead to wasteful unused pieces that are cut away.

06:39 Let's set the ratio of the length and width of a sheet of paper to 1 to x. The ratio of the sides of a rectangule that is cut to be half this size is also set at 1 to x. So let's find the value of x.
The answer is $x=\sqrt{2}$. Printing paper is cut according to this ratio. This is why standards for paper are so complicated.

07:07 (caption) A4-size paper

07:15 When construction models or maps are put together, this rule of similarity is indispensible. First the actual length and size of a building and every angle is measured.

07:28 Then the similarity ratio is determined. The figure is reduced and drawn with similar figures.

07:40 The ratio of similarity used in producing maps is called a scale.

07:30 (caption) ratio of similarity 1:100 → scale on a map

07:46 With the help of a map's scale, we can simply look at a map to determine the actual size of a region.

07:50 scale 1:100

07:53 For the flowerbed shaped like a triangle, we plug the width and height found on a map into the ratio of similarity.

08:07 With the results we get, we can determine the actual size of the flower bed through the formula for a triangle's area.

08:02 (caption)
actual distance
size of a triangle = ½ * width * height

08:14 When comparing the actual size to what's on the map, we know the ratio of the two areas equals the squared ratio of the numbers on the map's scale.

08:21 (caption)
flower bed size on the map
ratio of the sizes

08:37 scale of the map 1:100
ratio of the sizes 12:1002

08:42 Every polygon can be divided into triangles. The sum of those triangles' areas equals the size of the polygon.

08:50 In every polygon, the ratio of the size of similar figures equals the squares of the ratio of the corresponding lengths in each figure.

08:50 (caption)
ratio of a figure's size with the ratio of similitude m : n
= m2 : n2

09:00 Can this rule be applied to the ratio of volumes of similar three-dimensional figures?

09:08 (caption)
similarity ratio 2:3

09:09 Since the similarity ratio is 2 to 3, the radius of the cakes are 2x and 3x, and the heights are 2y and 3y, respectively.

09:21 Using these numbers, we can see the ratio of the volume of the cakes is…8 to 27. In other words, the result is the ratio of the cubes of the ratio of their respective lengths.

09:19 (caption)
volume of the cakes
ratio of the volumes = 23 : 33