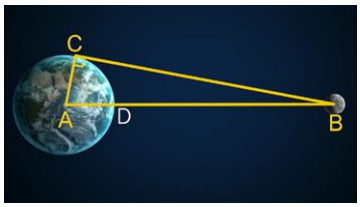

Mathematics, English for Sek I and Sek II

Mathematica - The Principles of Math

8. The Trigonometric Ratio, the Key to drawing the Skies

08:36 minutes

00:31 Do the priests know when it's going to rain?

00:36 (caption)
It's today.

00:39 They say it's today. As you may have noticed, the priests studied the stars yesterday to predict when it will rain.

00:48 Seeding and harvest time were determined according to the movement of the stars. Weather in the near future was also predicted.

00:56 For this reason, it was of the utmost importance for priests in ancient times to study the sky.

01:03 One concept of math that came from the studying of the stars and the need to read changes in the sky was the trigonometric ratio.

01:21 Ancient peoples believed that the stars in the sky were floating across a huge half sphere whose center was the Earth.

01:29 So they tried to read the movements of the stars by measuring the distance between them.

01:39 But there was no way to measure the stars' distance or the central angle of their arcs. They had no idea how to measure how many degrees were in the arc, which would tell them the distance between stars.

01:47 (caption)
length of arc
central angle

01:55 But then they came up with this idea.

02:01 Use your two arms to reach toward the stars. By using the angle between their own arms and the length of the arms, they could measure the distance between the stars.

02:10 But this had an obvious problem, too.

02:12 The calculated length between the stars would vary according to the length of different people's arms.

02:19 So their search for an answer finally led them to ratios.

02:24 (caption)
arms' length
measurement
ratio

02:27 If a central angle is the same no matter who is measuring it, the ratio between the arms' length and the distance between the tip of each hand would be constant.

02:43 With two arcs of the same sized angle, the ratio between the radius and the arc would be constant.

03:01 Astronomers put together a table after measuring these ratios.

03:06 Their efforts were recorded in the book Almagest, compiled by Greek astronomer Claudius Ptolemy and still passed down today.

03:08 (caption)
Almagest
written by Claudius Ptolemy around 150 BC

03:20 This became the original table for sine, the foundation for today's trigonometry.

03:36 (caption)
trigonometry ratio

03:36 Let's find out what the trigonometry ratio means.

03:44 This is the literal meaning.

03:43 (caption)
ratio of three sides of a right triangle

03:48 Two right triangles sharing one equal angle besides the right angle are similar.

03:54 Due to this point, the ratio of the three lines is constant between the two right triangles.

04:05 The ratio of the three sides, the basis of trigonometry, can be expressed like this:

04:08 (caption)
base, height, hypotenuse

04:15 …the ratio of the height over the hypotenuse, the ratio of the base over the hypotenuse, and the ratio of the height over the base.

04:15 (caption)
height/hypotenuse
base/hypotenuse
height/base

04:26 These three ratios are known as sine, cosine, and tangent. Each is used at different times.

04:44 (caption)
One sine and one cosine, please.

04:53 The Moon sometimes stays right over the horizon. Since the Earth is round, there are times when the Moon is right above our heads, at the same time and at the same longitude.

05:03 The Greek astronomer Hipparchos discovered this fact and found the solution to another mystery of space using trigonometry.

05:06 (caption)
Hipparchos (c. 160 BC – 250 BC)
Greek astronomer who organized spherical trigonometry

05:13 That was the distance between the Earth and the Moon.

05:17 To put it in plain words,…

05:20 When you see the Moon horizontally at point C on the Earth, draw a straight line to the Moon's center. Thus the tangent line from B to C is complete.

05:31 The angle between the tangent line and the Earth's radius is always a right angle.

05:41 At the same time, when the Moon is located perpendicularly from a line passing through point D, we draw a line from D to the Moon's center at B.

05:53 Since that line can be extended to the Earth's center, at point A, a giant right triangle ABC is formed whose hypotenuse is the distance between the Earth and Moon.

06:04 Now we know the value of cosine A and the Earth's radius, we can calculate the distance between the Earth and the Moon.

06:07 (caption)
cos A = Earth's radius/ distance between the Earth and the Moon

06:12 The ratio of the arc between points C and D over the Earth's circumference is equal to the ratio of the angle at point A over 360°. The size of angle A is easily obtained.

06:18 (caption)
arc length between C and D : Earth's circumference

06:33 With the help of trigonometric principles, Hipparchos was the first mathematician to come close to determining the distance between the Moon and the Earth.

06:37 (caption)
distance between Moon and Earth = Earth's radius/0.001658

06:52 This is the highest mountain in the world.

06:55 Do you know why it's named Mt Everest?

07:01 Those who knew about the usefulness of trigonometry began to apply ratios used in the sky to the land found on Earth.

07:08 Based on distances that were easily measured, they used trigonometry to determine distances that were not easily measured. After repeating the process, they calculated the distances they wanted to measure.

07:31 There was one person who accurately measured the distances across the Indian subcontinent using this system, called the triangulation method.

07:40 In order to honor this master of trigonometry, George Everest, the highest mountain in the world was named after him.

07:36 (caption)
Colonel Sir George Everest (1790 – 1866)
Welsh surveyor and geographer

07:54 Trigonometry began with the study of the sky.

07:59 It became useful in many fields, helping us measure things once thought not possible to measure.

08:10 And it is used quite a lot in daily life.

08:14 In the modern world, trigonometric functions are applied in a variety of areas, from music to psychology to construction.