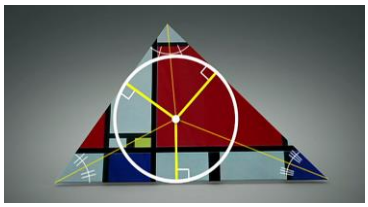

Mathematics, English for Sek I and Sek II

Mathematica - The Principles of Math

9. Finding Centers of Circles inscribed or circumscribed with Triangles

09:21 minutes

00:24 This is a treasure map.

00:27 It is said that the treasure lies buried at a spot the same distance from points A, B, and C.

00:33 So where does that mean the treasure is buried?

00:36 If you were on this island, how would you go and find it?

00:36 (caption) Equidistant (the same distance) from A, B, and C?

00:43 The answer involves finding the circumcenter of a triangle ABC.

00:49 Not sure what that means? Well, join us and find out.

01:00 A triangle is a flat figure consisting of the smallest number of segments of any polygon.

01:07 For centuries, the triangle was the most carefully researched subject of many mathematicians.
Probably every characteristic and principle of triangles has already been discovered.

01:17 The incenter and circumcenter of the triangle are two of its most important characteristics.

01:22 (caption) Every triangle has an incenter and a circumcenter.

01:23 So, what are the incenter and circumcenter of a triangle, and how do you go about locating them?

01:36 Let's fold this triangle so that we can divide each angle in half.

01:42 We take each line made from folding the angles in half and then see where they cross each other.

02:05 By folding it like this, we see that the three lines all meet at a point. That's the incenter of the triangle, which is defined as the center of a circle inscribed in the triangle.

02:11 (caption)
incenter of a triangle
found at the point inside a triangle where three lines bisect each angle meet

02:21 Now how about finding the circumcenter this time?

02:24 This time we fold the sides of the triangle in half to make the endpoints meet.

02:29 Since the triangle has three sides, we need to fold it three times.

02:48 You can see that three lines are created, each perpendicular to one side. They all meet at one point, the circumcenter of the triangle.

02:56 (caption)
circumcenter of a triangle
found at the point where three perpendicular lines bisecting to three lines meet

03:01 As you may have noticed, in the case of an acute triangle where all angles are less than 90°, the circumcenter is inside the triangle.

03:05 (caption)
circumcenter of an acute triangle (all angles less than 90°)
located inside the triangle

03:13 (caption)
circumcenter of a right triangle
located at the midpoint of the hypotenuse
circumcenter of an obtuse triangle (one angle greater than 90°)
located outside of the triangle

03:14 In a right triangle, the circumcenter is located at the midpoint of the hypotenuse, the longest side.
With an obtuse triangle where one angle is greater than 90°, the circumcenter is outside the triangle.

03:31 Now do you understand more about the incenter and circumcenter of a triangle? And as we mentioned below, the incenter and circumcenter of a triangle are closely connected with circles.

03:44 Take a look at what happens if we draw perpendicular lines from the incenter toward each side.
All these lines are equal in length.

03:58 That means that these lines become the radius of a circle inscribed in the triangle. In fact, the definition of the incenter is that it's the center of the inscribed circle.

03:58 (caption)
incenter
radius of the inscribed circle
inscribed circle

04:11 Similarly, the distance from the circumcenter to each of the endpoints of the triangle is equal.

04:17 That means this distance is the radius for a circle circumscribed around the triangle.

04:23 (caption)
circumcenter
radius of the circumscribed circle
circumscribed circle

04:33 The incenter and circumcenter of a triangle are explained in Elements of Geometry, written in the third century B.C. by Euclid, the father of geometry.

04:33 (caption)
Euclid (c. 300 BC)
Greek mathematician considered father of geometry

04:45 Euclid's Elements of Geometry

04:47 His famous book is the oldest geometry text in Europe, giving us definitions of the incenter and circumcenter of a triangle that we still use today.

05:02 If we're able to find the incenter of a circle inscribed inside a triangle, or the circumcenter of a circle circumscribed around a triangle, we can solve all sorts of problems in everyday life.

05:15 (caption) To make a triangular clock, where should we locate the clock's hands?

05:29 To make a triangular clock, we should draw a circle for the clock's hands that stays inside the triangle. We need to find that circle's center.

05:37 So that means we need to find the incenter of the triangle. Remember, the incenter is the meeting point of the three lines which divide each angle in half.

05:52 If we then take the distance from the incenter to each side and use it as a radius, we can draw an inscribed circle.

06:08 The inscribed circle is where the clock numbers would be located, and its center would be the incenter. That's where the clock hands would be put. Now it's complete.

06:26 (caption) In a village where all the homes form a circle, where should a park be built that is equal distance from each house?

06:31 Now let's make a park located the same distance from each of these houses. This time you need to know the circumcenter of the circle circumscribing a triangle.

06:42 Take the entire village and randomly select three homes. Draw a triangle using those houses as points.

06:49 If we draw a perpendicular line from each side, they will meet at a single point. That's the circumcenter of the triangle.

06:58 And then we connect the circumcenter with each end point. The three lines are equal and they form the radius of the circle circumscribing the triangle.

07:09 That means the circumcenter is located at the center of the village. That's the place to put the park so that it will be located at an equal distance from each house.

07:26 The incenter and the circumcenter of a triangle are used in a variety of areas in everyday life and in professional research.

07:41 Ancient relics are rarely found intact. But with artifacts like the famous face-tile from Korea's historic Shilla Dynasty, determining the circumcenter of a triangle in this shape can be used to restore its original look.

07:44 (caption)
Shilla Dynasty face tile
c. 7th century AD

07:58 We start by randomly taking three points on the edge of the relic and making a triangle. We then find the circumcenter of the triangle, which allows us to find the center of the relic itself. Based on this center point, it is possible to re-create the part that was lost.

08:24 Now let's go back to our treasure map in the very beginning of this episode.

08:28 Now we can find the treasure that was buried the same distance from points A, B, and C.

08:36 We find the circumcenter of triangle ABC and then connect it to the three endpoints. These lines are all equal to each other.

08:49 Bingo! The treasure is buried right beneath the circumcenter.

08:51 (caption) Hooray!

08:55 Maybe the next time you're out looking for treasure, you can remember this helpful tip!